

nag_regress_confid_interval (g02cbc)

1. Purpose

nag_regress_confid_interval (g02cbc) performs a simple linear regression with or without a constant term. The data is optionally weighted, and confidence intervals are calculated for the predicted and average values of y at a given x .

2. Specification

```
#include <nag.h>
#include <nagg02.h>

void nag_regress_confid_interval(Nag_SumSquare mean, Integer n,
    double x[], double y[], double wt[], double clm, double clp,
    double yhat[], double yml[], double ymu[], double yl[], double yu[],
    double h[], double res[], double *rms, NagError *fail)
```

3. Description

This function fits a straight line model of the form,

$$E(y) = a + bx$$

where $E(y)$ is the expected value of the variable y , to the data points

$$(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n),$$

such that

$$y_i = a + bx_i + e_i, \quad i = 1, 2, \dots, n,$$

where the e_i values are independent random errors. The i th data point may have an associated weight w_i . The values of a and b are estimated by minimizing $\sum w_i e_i^2$ (if the weights option is not selected then $w_i = 1.0$).

The fitted values \hat{y}_i are calculated using

$$\hat{y}_i = \hat{a} + \hat{b}x_i$$

where

$$\hat{a} = \bar{y} - b\bar{x} \quad \hat{b} = \frac{\sum w_i(x_i - \bar{x})(y_i - \bar{y})}{\sum w_i(x_i - \bar{x})^2}$$

and the weighted means \bar{x} and \bar{y} are given by

$$\bar{y} = \frac{\sum w_i y_i}{\sum w_i} \quad \text{and} \quad \bar{x} = \frac{\sum w_i x_i}{\sum w_i}.$$

The residuals of the regression are calculated using

$$res_i = y_i - \hat{y}_i$$

and the residual mean square about the regression rms , is determined using

$$rms = \frac{\sum w_i (y_i - \hat{y}_i)^2}{df}$$

where df (the number of degrees of freedom) has the following values

$$\begin{aligned} df &= \sum w_i - 2 & \text{where } \mathbf{mean} &= \mathbf{Nag_AboutMean} \\ df &= \sum w_i - 1 & \text{where } \mathbf{mean} &= \mathbf{Nag_AboutZero}. \end{aligned}$$

Note: the weights should be scaled to give the required degrees of freedom.

The function calculates predicted y estimates for a value of x , x_i^* , is given by

$$y_i^* = \hat{a} + \hat{b}x_i^*$$

this prediction has a standard error

$$serr_pred = \sqrt{rms} \sqrt{1 + \frac{1}{\sum w_i} + \frac{(x_i^* - \bar{x})^2}{\sum w_i (x_i - \bar{x})^2}}.$$

The $(1 - \alpha)$ confidence interval for this estimation of y is given by

$$y_i^* \pm t_{df}(1 - \alpha/2).serr_pred$$

where $t_{df}(1 - \alpha/2)$ refers to the $(1 - \alpha/2)$ point of the t distribution with df degrees of freedom (e.g. when $df = 20$ and $\alpha = 0.1$, $t_{20}(0.95) = 2.086$). If the user specifies the probability $clp = 0.9(\alpha = 0.1)$ then the lower limit of this interval is

$$y_i^l = y_i^* - t_{df}(0.95).serr_pred$$

and the upper limit is

$$y_i^u = y_i^* + t_{df}(0.95).serr_pred.$$

The mean value of y at x_i is estimated by the fitted value \hat{y}_i . This has a standard error of

$$serr_arg = \sqrt{rms} \sqrt{\frac{1}{\sum w_i} + \frac{(x_i - \bar{x})^2}{\sum w_i (x_i - \bar{x})^2}}$$

and a $(1 - \alpha)$ confidence interval is given by

$$\hat{y}_i \pm t_{df}(1 - \alpha/2).serr_arg.$$

For example, if the user specifies the probability $clm = 0.6(\alpha = 0.4)$ then the lower limit of this interval is

$$y_i^{ml} = \hat{y}_i - t_{df}(0.8).serr_arg$$

and the upper limit is

$$y_i^{mu} = \hat{y}_i + t_{df}(0.8).serr_arg.$$

The leverage, h_i , is a measure of the influence a value x_i has on the fitted line at that point, \hat{y}_i . The leverage is given by

$$h_i = \frac{w_i}{\sum w_i} + \frac{w_i(x_i - \bar{x})^2}{\sum w_i (x_i - \bar{x})^2}$$

so it can be seen that

$$serr_arg = \sqrt{rms} \sqrt{h_i/w_i}$$

and $serr_pred = \sqrt{rms} \sqrt{1 + h_i/w_i}$

Similar formulae can be derived for the case when the line goes through the origin, that is $a = 0$.

4. Parameters

mean

Input: indicates whether nag_regress_confid_interval is to include a constant term in the regression.

If **mean** = **Nag_AboutMean**, the constant term, a , is included.

If **mean** = **Nag_AboutZero**, the constant term, a , is not included, i.e., $a = 0$.

Constraint: **mean** = **Nag_AboutMean** or **Nag_AboutZero**.

n

Input: the number of observations, **n**.

Constraint: if **mean** = **Nag_AboutMean** $n \geq 2$. if **mean** = **Nag_AboutZero** $n \geq 1$.

x[n]

Input: observations on the independent variable, x .

Constraint: all the values of x must not be identical.

- y[n]**
Input: observations on the dependent variable, y .
- wt[n]**
Input: if weighted estimates are required then **wt** must contain the weights to be used in the weighted regression. Otherwise **wt** need not be defined and may be set to the null pointer **NULL**, i.e. (double *)0.
Usually **wt**[$i - 1$] will be an integral value corresponding to the number of observations associated with the i th data point, or zero if the i th data point is to be ignored. The sum of the weights therefore represents the effective total number of observations used to create the regression line.
If **wt** = **NULL**, then the effective number of observations is n .
Constraint: **wt** = **NULL** or **wt**[$i - 1$] ≥ 0.0 , for $i = 1, \dots, n$.
- clm**
Input: the confidence level for the confidence intervals for the mean.
Constraint: $0.0 < \text{clm} < 1.0$.
- clp**
Input: the confidence level for the prediction intervals.
Constraint: $0.0 < \text{clp} < 1.0$.
- yhat[n]**
Output: the fitted values, \hat{y}_i .
- yml[n]**
Output: **yml**[$i - 1$] contains the lower limit of the confidence interval for the regression line at **x**[$i - 1$].
- ymu[n]**
Output: **ymu**[$i - 1$] contains the upper limit of the confidence interval for the regression line at **x**[$i - 1$].
- yl[n]**
Output: **yl**[$i - 1$] contains the lower limit of the confidence interval for the individual y value at **x**[$i - 1$].
- yu[n]**
Output: **yu**[$i - 1$] contains the upper limit of the confidence interval for the individual y value at **x**[$i - 1$].
- h[n]**
Output: the leverage of each observation on the regression.
- res[n]**
Output: the residuals of the regression.
- rms**
Output: the residual mean square about the regression.
- fail**
The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

NE_BAD_PARAM

On entry, parameter **mean** had an illegal value.

NE_INT_ARG_LT

On entry, **n** must not be less than 1: **n** = $\langle \text{value} \rangle$

if **mean** = **Nag_AboutZero**.

On entry, **n** must not be less than 2: **n** = $\langle \text{value} \rangle$

if **mean** = **Nag_AboutMean**.

NE_REAL_ARG_GE

On entry, **clm** must not be greater than or equal to 1.0: **clm** = $\langle \text{value} \rangle$.

On entry, **clp** must not be greater than or equal to 1.0: **clp** = $\langle \text{value} \rangle$.

NE_REAL_ARG_LE

On entry, **clm** must not be less than or equal to 0.0: **clm** = *<value>*.

On entry, **clp** must not be less than or equal to 0.0: **clp** = *<value>*.

NE_NEG_WEIGHT

On entry, at least one of the weights is negative.

NE_WT_LOW

On entry, **wt** must contain at least 1 positive element if **mean** = **Nag_AboutZero** or at least 2 positive elements if **mean** = **Nag_AboutMean**.

NE_X_IDEN

On entry, all elements of **x** are equal.

NE_SW_LOW

On entry, the sum of elements of **wt** must be greater than 1.0 if **mean** = **Nag_AboutZero** and 2.0 if **mean** = **Nag_AboutMean**.

NW_RMS_EQ_ZERO

Residual mean sum of squares is zero, i.e., a perfect fit was obtained.

6. Further Comments

None.

6.1. Accuracy

The computations are believed to be stable.

6.2. References

Snedecor G W and Cochran W G (1967) *Statistical Methods*. (6th Edn) Iowa State University Press.

7. See Also

nag_simple_linear_regression (g02cac)

8. Example

A program to calculate the fitted value of *y* and the upper and lower limits of the confidence interval for the regression line as well as the individual *y* values.

8.1. Program Text

```
/* nag_regress_confid_interval(g02cbc) Example Program
 *
 * Copyright 1994 Numerical Algorithms Group.
 *
 * Mark 3, 1994.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg02.h>

#define NMAX 10
```

```

main()
{
    Nag_SumSquare mean;
    Integer n;
    double x[NMAX], y[NMAX], wt[NMAX];
    double clm, clp;
    double yhat[NMAX], yml[NMAX], ymu[NMAX], yl[NMAX], yu[NMAX], h[NMAX],
    res[NMAX], rms;
    Integer i;
    char m, w;

    Vprintf("g02cbc Example Program Results\n");
    /* Skip heading in data file */
    Vscanf("%*[^\\n]");
    Vscanf("%ld\n",&n);
    if (n>=1 && n<= NMAX)
    {
        Vscanf("%lf%lf\n",&clm,&clp);
        Vscanf(" %c %c\n",&m,&w);
        if (m == 'm' || m == 'M')
            mean = Nag_AboutMean;
        else if (m == 'z' || m == 'Z')
            mean = Nag_AboutZero;
        if (w == 'w' || w == 'W')
            for (i=0; i<n; i++)
                Vscanf("%lf%lf%lf\n",&x[i],&y[i],&wt[i]);
        else
            for (i = 0; i < n; ++i)
                Vscanf("%lf%lf\n",&x[i],&y[i]);

        g02cbc(mean, n, x, y, wt, clm, clp, yhat, yml, ymu, yl, yu, h, res,
            &rms, NAGERR_DEFAULT);

        Vprintf ("\ni          yhat[i]    yml[i]    ymu[i]    yl[i]    yu[i]\n\n");
        for (i=0; i < n; ++i) {
            Vprintf("%ld %10.2f %10.2f", i, yhat[i], yml[i]);
            Vprintf("%10.2f %10.2f %10.2f\n",ymu[i], yl[i], yu[i]);
        }
    }
    else
    {
        Vfprintf(stderr, "n is out of range:\n
n = %-3ld\n",n);
        exit(EXIT_FAILURE);
    }
    exit(EXIT_SUCCESS);
}

```

8.2. Program Data

```

g02cbc Example Program Data
9
0.95 0.95
mw
1.0 4.0 1.0
2.0 4.0 2.0
4.0 5.1 1.0
2.0 4.0 1.0
2.0 6.0 1.0
3.0 5.2 1.0
7.0 9.1 1.0
4.0 2.0 1.0
2.0 4.1 1.0

```

8.3. Program Results

g02cbc Example Program Results

i	yhat[i]	yml[i]	ymu[i]	yl[i]	yu[i]
0	3.47	1.76	5.18	-0.46	7.40
1	4.14	2.87	5.42	0.38	7.90
2	5.49	4.15	6.84	1.71	9.27
3	4.14	2.87	5.42	0.38	7.90
4	4.14	2.87	5.42	0.38	7.90
5	4.82	3.70	5.94	1.11	8.53
6	7.52	4.51	10.53	2.87	12.16
7	5.49	4.15	6.84	1.71	9.27
8	4.14	2.87	5.42	0.38	7.90
