

## nag\_mv\_discrim\_group (g03dcc)

### 1. Purpose

**nag\_mv\_discrim\_group (g03dcc)** allocates observations to groups according to selected rules. It is intended for use after `nag_mv_discrim (g03dac)`.

### 2. Specification

```
#include <nag.h>
#include <nagg03.h>

void nag_mv_discrim_group(Nag_DiscrimMethod type, Nag_GroupCovars equal,
    Nag_PriorProbability priors, Integer nvar,
    Integer ng, Integer nig[], double gmean[], Integer tdg,
    double gc[], double det[], Integer nobs, Integer m,
    Integer isx[], double x[], Integer tdx,
    double prior[], double p[], Integer tdp, Integer iag[],
    Boolean atiq, double ati[], NagError *fail)
```

### 3. Description

Discriminant analysis is concerned with the allocation of observations to groups using information from other observations whose group membership is known,  $X_t$ ; these are called the training set. Consider  $p$  variables observed on  $n_g$  populations or groups. Let  $\bar{x}_j$  be the sample mean and  $S_j$  the within-group variance-covariance matrix for the  $j$ th group; these are calculated from a training set of  $n$  observations with  $n_j$  observations in the  $j$ th group, and let  $x_k$  be the  $k$ th observation from the set of observations to be allocated to the  $n_g$  groups. The observation can be allocated to a group according to a selected rule. The allocation rule or discriminant function will be based on the distance of the observation from an estimate of the location of the groups, usually the group means. A measure of the distance of the observation from the  $j$ th group mean is given by the Mahalanobis distance,  $D_{kj}^2$ :

$$D_{kj}^2 = (x_k - \bar{x}_j)^T S_j^{-1} (x_k - \bar{x}_j). \quad (1)$$

If the pooled estimate of the variance-covariance matrix  $S$  is used rather than the within-group variance-covariance matrices, then the distance is:

$$D_{kj}^2 = (x_k - \bar{x}_j)^T S^{-1} (x_k - \bar{x}_j). \quad (2)$$

Instead of using the variance-covariance matrices  $S$  and  $S_j$ , `nag_mv_discrim_group` uses the upper triangular matrices  $R$  and  $R_j$  supplied by `nag_mv_discrim (g03dac)` such that  $S = R^T R$  and  $S_j = R_j^T R_j$ .  $D_{kj}^2$  can then be calculated as  $z^T z$  where  $R_j z = (x_k - \bar{x}_j)$  or  $Rz = (x_k - \bar{x}_j)$  as appropriate.

In addition to the distances, a set of prior probabilities of group membership,  $\pi_j$ , for  $j = 1, 2, \dots, n_g$ , may be used, with  $\sum \pi_j = 1$ . The prior probabilities reflect the user's view as to the likelihood of the observations coming from the different groups. Two common cases for prior probabilities are  $\pi_1 = \pi_2 = \dots = \pi_{n_g}$ , that is, equal prior probabilities, and  $\pi_j = n_j/n$ , for  $j = 1, 2, \dots, n_g$ , that is, prior probabilities proportional to the number of observations in the groups in the training set.

`nag_mv_discrim_group` uses one of four allocation rules. In all four rules the  $p$  variables are assumed to follow a multivariate Normal distribution with mean  $\mu_j$  and variance-covariance matrix  $\Sigma_j$  if the observation comes from the  $j$ th group. The different rules depend on whether or not the within-group variance-covariance matrices are assumed equal, i.e.,  $\Sigma_1 = \Sigma_2 = \dots = \Sigma_{n_g}$ , and whether a predictive or estimative approach is used. If  $p(x_k | \mu_j, \Sigma_j)$  is the probability of observing the observation  $x_k$  from group  $j$ , then the posterior probability of belonging to group  $j$  is:

$$p(j | x_k, \mu_j, \Sigma_j) \propto p(x_k | \mu_j, \Sigma_j) \pi_j. \quad (3)$$

In the estimative approach, the parameters  $\mu_j$  and  $\Sigma_j$  in (3) are replaced by their estimates calculated from  $X_t$ . In the predictive approach, a non-informative prior distribution is used for

the parameters and a posterior distribution for the parameters,  $p(\mu_j, \Sigma_j | X_t)$ , is found. A predictive distribution is then obtained by integrating  $p(j|x_k, \mu_j, \Sigma_j)p(\mu_j, \Sigma_j | X)$  over the parameter space. This predictive distribution then replaces  $p(x_k | \mu_j, \Sigma_j)$  in (3). See Aitchison and Dunsmore (1975), Aitchison *et al.* (1977) and Moran and Murphy (1979) for further details.

The observation is allocated to the group with the highest posterior probability. Denoting the posterior probabilities,  $p(j|x_k, \mu_j, \Sigma_j)$ , by  $q_j$ , the four allocation rules are:

- (i) Estimative with equal variance-covariance matrices – Linear Discrimination.

$$\log q_j \propto -\frac{1}{2}D_{kj}^2 + \log \pi_j$$

- (ii) Estimative with unequal variance-covariance matrices – Quadratic Discrimination.

$$\log q_j \propto -\frac{1}{2}D_{kj}^2 + \log \pi_j - \frac{1}{2} \log |S_j|$$

- (iii) Predictive with equal variance-covariance matrices.

$$q_j^{-1} \propto ((n_j + 1)/n_j)^{p/2} \{1 + [n_j / ((n - n_g)(n_j + 1))] D_{kj}^2\}^{(n+1-n_g)/2}$$

- (iv) Predictive with unequal variance-covariance matrices.

$$q_j^{-1} \propto C \{((n_j^2 - 1)/n_j) |S_j|\}^{p/2} \{1 + (n_j / (n_j^2 - 1)) D_{kj}^2\}^{n_j/2}$$

where

$$C = \frac{\Gamma(\frac{1}{2}(n_j - p))}{\Gamma(\frac{1}{2}n_j)}$$

In the above the appropriate value of  $D_{kj}^2$  from (1) or (2) is used. The values of the  $q_j$  are standardized so that,

$$\sum_{j=1}^{n_g} q_j = 1.$$

Moran and Murphy (1979) show the similarity between the predictive methods and methods based upon likelihood ratio tests.

In addition to allocating the observation to a group, nag\_mv\_discrim\_group computes an atypicality index,  $I_j(x_k)$ . This represents the probability of obtaining an observation more typical of group  $j$  than the observed  $x_k$  (see Aitchison and Dunsmore (1975) and Aitchison *et al.* (1977)). The atypicality index is computed as:

$$I_j(x_k) = P(B \leq z : \frac{1}{2}p, \frac{1}{2}(n_j - d))$$

where  $P(B \leq \beta : a, b)$  is the lower tail probability from a beta distribution where, for unequal within-group variance-covariance matrices,

$$z = D_{kj}^2 / (D_{kj}^2 + (n_j^2 - 1)/n_j),$$

and for equal within-group variance-covariance matrices,

$$z = D_{kj}^2 / (D_{kj}^2 + (n - n_g)(n_j - 1)/n_j).$$

If  $I_j(x_k)$  is close to 1 for all groups it indicates that the observation may come from a grouping not represented in the training set. Moran and Murphy (1979) provide a frequentist interpretation of  $I_j(x_k)$ .

#### 4. Parameters

##### type

Input: indicates whether the estimative or predictive approach is to be used.

If **type** = **Nag\_DiscrimEstimate**, the estimative approach is used.

If **type** = **Nag\_DiscrimPredict**, the predictive approach is used.

Constraint: **type** = **Nag\_DiscrimEstimate** or **Nag\_DiscrimPredict**.

##### equal

Input: indicates whether or not the within-group variance-covariance matrices are assumed to be equal and the pooled variance-covariance matrix used.

If **equal** = **Nag\_EqualCovar**, the within-group variance-covariance matrices are assumed equal and the matrix  $R$  stored in the first  $p(p+1)/2$  elements of **gc** is used.

If **equal** = **Nag\_NotEqualCovar**, the within-group variance-covariance matrices are assumed to be unequal and the matrices  $R_i$ , for  $i = 1, 2, \dots, n_g$ , stored in the remainder of **gc** are used.

Constraint: **equal** = **Nag\_EqualCovar** or **Nag\_NotEqualCovar**.

##### priors

Input: indicates the form of the prior probabilities to be used.

If **priors** = **Nag\_EqualPrior**, equal prior probabilities are used.

If **priors** = **Nag\_GroupSizePrior**, prior probabilities proportional to the group sizes in the training set,  $n_j$ , are used.

If **priors** = **Nag\_UserPrior**, the prior probabilities are input in **prior**.

Constraint: **priors** = **Nag\_EqualPrior**, **Nag\_GroupSizePrior** or **Nag\_UserPrior**.

##### nvar

Input: the number of variables,  $p$ , in the variance-covariance matrices as specified to `nag_mv_discrim` (g03dac).

Constraint: **nvar**  $\geq 1$ .

##### ng

Input: the number of groups,  $n_g$ .

Constraint: **ng**  $\geq 2$ .

##### nig[ng]

Input: the number of observations in each group training set,  $n_j$ .

Constraints:

If **equal** = **Nag\_EqualCovar**, **nig**[ $j-1$ ]  $> 0$ , for  $j = 1, 2, \dots, n_g$  and  $\sum_{j=1}^{n_g} \mathbf{nig}[j-1] > \mathbf{ng} + \mathbf{nvar}$ .

If **equal** = **Nag\_NotEqualCovar**, **nig**[ $j-1$ ]  $> \mathbf{nvar}$ , for  $j = 1, 2, \dots, n_g$ .

##### gmean[ng][tdg]

Input: the  $j$ th row of **gmean** contains the means of the  $p$  variables for the  $j$ th group, for  $j = 1, 2, \dots, n_g$ . These are returned by `nag_mv_discrim` (g03dac).

##### tdg

Input: the last dimension of the array **gmean** as declared in the calling program.

Constraint: **tdg**  $\geq \mathbf{nvar}$

##### gc[(ng+1)\*nvar\*(nvar+1)/2]

Input: the first  $p(p+1)/2$  elements of **gc** should contain the upper triangular matrix  $R$  and the next  $n_g$  blocks of  $p(p+1)/2$  elements should contain the upper triangular matrices  $R_j$ .

All matrices must be stored packed by column. These matrices are returned by `nag_mv_discrim` (g03dac). If **equal**=**Nag\_EqualCovar** only the first  $p(p+1)/2$  elements are referenced, if **equal** = **Nag\_NotEqualCovar** only the elements  $p(p+1)/2$  to  $(n_g+1)p(p+1)/2-1$  are referenced.

Constraints:

If **equal** = **Nag\_EqualCovar**, the diagonal elements of  $R$  must be  $\neq 0.0$ ,

If **equal** = **Nag\_NotEqualCovar**, the diagonal elements of the  $R_j$  must be  $\neq 0.0$ , for  $j = 1, 2, \dots, n_g$ .

**det[ng]**

Input: if **equal** = **Nag\_NotEqualCovar** the logarithms of the determinants of the within-group variance-covariance matrices as returned by nag\_mv\_discrim (g03dac). Otherwise **det** is not referenced.

**nobs**

Input: the number of observations in **x** which are to be allocated.

Constraint: **nobs**  $\geq$  1.

**m**

Input: the number of variables in the data array **x**.

Constraint: **m**  $\geq$  **nvar**.

**isx[m]**

Input: **isx**[ $l-1$ ] indicates if the  $l$ th variable in **x** is to be included in the distance calculations. If **isx**[ $l-1$ ]  $>$  0 the  $l$ th variable is included, for  $l = 1, 2, \dots, \mathbf{m}$ ; otherwise the  $l$ th variable is not referenced.

Constraint: **isx**[ $l-1$ ]  $>$  0 for **nvar** values of  $l$ .

**x[nobs][tdx]**

Input: **x**[ $k-1$ ][ $l-1$ ] must contain the  $k$ th observation for the  $l$ th variable, for  $k = 1, 2, \dots, \mathbf{nobs}$ ;  $l = 1, 2, \dots, \mathbf{m}$ .

**tdx**

Input: the last dimension of the array **x** as declared in the calling program.

Constraint: **tdx**  $\geq$  **m**.

**prior[ng]**

Input: if **priors** = **Nag\_UserPrior**, the prior probabilities for the  $n_g$  groups.

Constraint: if **priors** = **Nag\_UserPrior**, then **prior**[ $j-1$ ]  $>$  0.0 for  $j = 1, 2, \dots, n_g$  and  $|1 - \sum_{j=1}^{n_g} \mathbf{prior}[j-1]| \leq 10 \times \mathbf{machine\ precision}$ .

Output: if **priors** = **Nag\_GroupSizePrior**, the computed prior probabilities in proportion to group sizes for the  $n_g$  groups. If **priors** = **Nag\_UserPrior**, the input prior probabilities will be unchanged, and if **priors** = **Nag\_EqualPrior**, **prior** is not set.

**p[nobs][tdp]**

Output: **p**[ $k-1$ ][ $j-1$ ] contains the posterior probability  $p_{kj}$  for allocating the  $k$ th observation to the  $j$ th group, for  $k = 1, 2, \dots, \mathbf{nobs}$ ;  $j = 1, 2, \dots, n_g$ .

**tdp**

Input: the last dimension of the array **p** and **ati** as declared in the calling program.

Constraint: **tdp**  $\geq$  **ng**.

**iag[nobs]**

Output: the groups to which the observations have been allocated.

**atiq**

Input: **atiq** must be **TRUE** if atypicality indices are required. If **atiq** is **FALSE**, the array **ati** is not set.

**ati[nobs][tdp]**

Output: if **atiq** is **TRUE**, **ati**[ $k-1$ ][ $j-1$ ] will contain the atypicality index for the  $k$ th observation with respect to the  $j$ th group, for  $k = 1, 2, \dots, \mathbf{nobs}$ ;  $j = 1, 2, \dots, n_g$ . If **atiq** is **FALSE**, **ati** is not set.

**fail**

The NAG error parameter, see the Essential Introduction to the NAG C Library.

## 5. Error Indications and Warnings

### NE\_BAD\_PARAM

On entry, parameter **type** had an illegal value.  
 On entry, parameter **equal** had an illegal value.  
 On entry, parameter **priors** had an illegal value.

### NE\_INT\_ARG\_LT

On entry, **nvar** must not be less than 1: **nvar** =  $\langle value \rangle$ .  
 On entry, **ng** must not be less than 2: **ng** =  $\langle value \rangle$ .  
 On entry, **nobs** must not be less than 1: **nobs** =  $\langle value \rangle$ .

### NE\_2\_INT\_ARG\_LT

On entry, **m** =  $\langle value \rangle$  while **nvar** =  $\langle value \rangle$ .  
 These parameters must satisfy  $\mathbf{m} \geq \mathbf{nvar}$ .  
 On entry, **tdx** =  $\langle value \rangle$  while **m** =  $\langle value \rangle$ .  
 These parameters must satisfy  $\mathbf{tdx} \geq \mathbf{m}$ .  
 On entry, **tdp** =  $\langle value \rangle$  while **ng** =  $\langle value \rangle$ .  
 These parameters must satisfy  $\mathbf{tdp} \geq \mathbf{ng}$ .  
 On entry, **tdg** =  $\langle value \rangle$  while **nvar** =  $\langle value \rangle$ .  
 These parameters must satisfy  $\mathbf{tdg} \geq \mathbf{nvar}$ .

### NE\_VAR\_INCL\_INDICATED

The number of variables, **nvar** in the analysis =  $\langle value \rangle$ , while number of variables included in the analysis via array **isx** =  $\langle value \rangle$ .  
 Constraint: these two numbers must be the same.

### NE\_INTARR

On entry, **nig**[ $\langle value \rangle$ ] =  $\langle value \rangle$ .  
 Constraint: **nig**[ $i - 1$ ] > 0,  $i = 1, 2, \dots, \mathbf{ng}$  when **equal** = Nag\_EqualCovar.

### NE\_INTARR\_INT

On entry, **nig**[ $\langle value \rangle$ ] =  $\langle value \rangle$ , **nvar** =  $\langle value \rangle$ .  
 Constraint: **nig**[ $i - 1$ ] > **nvar**,  $i = 1, 2, \dots, \mathbf{ng}$  when **equal** = Nag\_NotEqualCovar.

### NE\_REALARR

On entry, **prior**[ $\langle value \rangle$ ] =  $\langle value \rangle$ .  
 Constraint: **prior**[ $j - 1$ ] > 0,  $j = 1, 2, \dots, \mathbf{ng}$  when **priors** = Nag\_UserPrior.

### NE\_PRIOR\_SUM

On entry,  $\sum_{j=1}^{\mathbf{ng}} \mathbf{prior}[j - 1] = \langle value \rangle$ .  
 Constraint:  $\sum_{j=1}^{\mathbf{ng}} \mathbf{prior}[j - 1]$  must be within  $10 \times \mathit{machine\ precision}$  of 1 when **priors** = Nag\_UserPrior.

### NE\_GROUP\_SUM

On entry, the  $\sum_{j=1}^{\mathbf{ng}} \mathbf{nig}[j - 1] = \langle value \rangle$ , **ng** =  $\langle value \rangle$ , **nvar** =  $\langle value \rangle$ .  
 Constraint:  $\sum_{j=1}^{\mathbf{ng}} \mathbf{nig}[j - 1] > \mathbf{ng} + \mathbf{nvar}$  when **equal** = Nag\_EqualCovar.

### NE\_DIAG\_0\_COND

A diagonal element of R is zero when **equal** = Nag\_EqualCovar.

### NE\_DIAG\_0\_J\_COND

A diagonal element of R is zero for some  $j$ , when **equal** = Nag\_NotEqualCovar

### NE\_ALLOC\_FAIL

Memory allocation failed.

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function.  
 Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

## 6. Further Comments

The distances  $D_{kj}^2$  can be computed using nag\_mv\_discrim\_mahaldist (g03dbc) if other forms of discrimination are required.

**6.1. Accuracy**

The accuracy of the returned posterior probabilities will depend on the accuracy of the input  $R$  or  $R_j$  matrices. The atypicality index should be accurate to four significant places.

**6.2. References**

- Aitchison J and Dunsmore I R (1975) *Statistical Prediction Analysis* Cambridge.  
 Aitchison J, Habbema J D F and Kay J W (1977) A critical comparison of two methods of statistical discrimination *Appl. Statist.* **26** 15–25.  
 Kendall M G and Stuart A (1976) *The Advanced Theory of Statistics (Volume 3)* Griffin (3rd Edition).  
 Krzanowski W J (1990) *Principles of Multivariate Analysis* Oxford University Press.  
 Moran M A and Murphy B J (1979) A closer look at two alternative methods of statistical discrimination *Appl. Statist.* **28** 223–232.  
 Morrison D F (1967) *Multivariate Statistical Methods* McGraw-Hill.

**7. See Also**

nag\_mv\_discrim\_mahaldist (g03dbc)  
 nag\_mv\_discrim (g03dac)

**8. Example**

The data, taken from Aitchison and Dunsmore (1975), is concerned with the diagnosis of three ‘types’ of Cushing’s syndrome. The variables are the logarithms of the urinary excretion rates (mg/24hr) of two steroid metabolites. Observations for a total of 21 patients are input and the group means and  $R$  matrices are computed by nag\_mv\_discrim (g03dac). A further six observations of unknown type are input and allocations made using the predictive approach and under the assumption that the within-group covariance matrices are not equal. The posterior probabilities of group membership,  $q_j$ , and the atypicality index are printed along with the allocated group. The atypicality index shows that observations 5 and 6 do not seem to be typical of the three types present in the initial 21 observations.

**8.1. Program Text**

```

/* nag_mv_discrim_group (g03dcc) Example Program.
 *
 * Copyright 1998 Numerical Algorithms Group.
 *
 * Mark 5, 1998.
 */
#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagg03.h>

#define NMAX 21
#define MMAX 2
#define GPMAX 3

main()
{
  double stat;
  double ati[NMAX][GPMAX], det[GPMAX],
  gc[(GPMAX+1)*MMAX*(MMAX+1)/2], gmean[GPMAX][MMAX],
  p[NMAX][GPMAX], prior[GPMAX],
  wt[NMAX], x[NMAX][MMAX];
  double df;
  double sig;
  double *wtptr=0;

  Integer nobs, nvar;
  Integer i, j, m, n;

```

```

Integer iag[NMAX], ing[NMAX], isx[MMAX],
nig[GPMAX];
Integer ng;
Integer tdgmean=MMAX, tdp=GPMAX, tdx=MMAX;

Boolean atiq = TRUE;

char char_type[2];
char char_equal[2];
char weight[2];

Nag_DiscrimMethod type;
Nag_GroupCovars equal;

Vprintf("g03dcc Example Program Results\n\n");

/* Skip headings in data file */
Vscanf("%*[^\\n]");

Vscanf("%ld",&n);
Vscanf("%ld",&m);
Vscanf("%ld",&nvar);
Vscanf("%ld",&ng);
Vscanf("%s",weight);

if (n <= NMAX && m <= MMAX)
{
  if (*weight == 'W')
  {
    for (i = 0; i < n; ++i)
    {
      for (j = 0; j < m; ++j)
        Vscanf("%lf",&x[i][j]);
      Vscanf("%ld",&ing[i]);
      Vscanf("%lf",&wt[i]);
    }
    wtptr = wt;
  }
  else
  {
    for (i = 0; i < n; ++i)
    {
      for (j = 0; j < m; ++j)
        Vscanf("%lf",&x[i][j]);
      Vscanf("%ld",&ing[i]);
    }
  }
  for (j = 0; j < m; ++j)
    Vscanf("%ld",&isx[j]);

  g03dac(n, m, (double *)x, tdx, isx, nvar, ing, ng, wtptr, nig,
        (double *)gmean, tdgmean, det, gc, &stat, &df, &sig, NAGERR_DEFAULT);
  Vscanf("%ld",&nobs);
  Vscanf("%s",char_equal);
  Vscanf("%s",char_type);
  if (nobs <= MMAX)
  {
    for (i = 0; i < nobs; ++i)
    {
      for (j = 0; j < m; ++j)
      {
        Vscanf("%lf",&x[i][j]);
      }
    }

    if (*char_type == 'E')
      type = Nag_DiscrimEstimate;
    else if (*char_type == 'P')
      type = Nag_DiscrimPredict;
  }
}

```

```

if (*char_equal == 'E')
    equal = Nag_EqualCovar;
else if (*char_equal == 'U')
    equal = Nag_NotEqualCovar;

g03dcc(type, equal, Nag_EqualPrior, nvar, ng, nig, (double *)gmean,
        tdgmean, gc, det, nobs, m, isx, (double *)x, tdx, prior, (double *)p,
        tdp, iag, atiq, (double *)ati, NAGERR_DEFAULT);

Vprintf("\n");
Vprintf("    Obs          Posterior          Allocated ");
Vprintf("          Atypicality ");
Vprintf("\n");
Vprintf("          probabilities    to group    index ");
Vprintf("\n");
Vprintf("\n");
for (i = 0; i < nobs; ++i)
{
    Vprintf(" %6ld      ", i+1);
    for (j = 0; j < ng; ++j)
    {
        Vprintf("%6.3f", p[i][j]);
    }
    Vprintf(" %6ld      ", iag[i]);
    for (j = 0; j < ng; ++j)
    {
        Vprintf("%6.3f", ati[i][j]);
    }
    Vprintf("\n");
}
}

exit(EXIT_SUCCESS);
}
else
{
    Vprintf("Incorrect input value of n or m.\n");
    exit(EXIT_FAILURE);
}
}

```

## 8.2. Program Data

```

g03dcc Example Program Data
21 2 2 3 U
1.1314    2.4596    1
1.0986    0.2624    1
0.6419   -2.3026    1
1.3350   -3.2189    1
1.4110    0.0953    1
0.6419   -0.9163    1
2.1163    0.0000    2
1.3350   -1.6094    2
1.3610   -0.5108    2
2.0541    0.1823    2
2.2083   -0.5108    2
2.7344    1.2809    2
2.0412    0.4700    2
1.8718   -0.9163    2
1.7405   -0.9163    2
2.6101    0.4700    2
2.3224    1.8563    3
2.2192    2.0669    3
2.2618    1.1314    3
3.9853    0.9163    3
2.7600    2.0281    3
1          1
6 U P
1.6292   -0.9163
2.5572    1.6094
2.5649   -0.2231

```



0.9555 -2.3026  
3.4012 -2.3026  
3.0204 -0.2231

**8.3. Program Results**

g03dcc Example Program Results

Obs	Posterior probabilities			Allocated to group	Atypicality index		
1	0.094	0.905	0.002	2	0.596	0.254	0.975
2	0.005	0.168	0.827	3	0.952	0.836	0.018
3	0.019	0.920	0.062	2	0.954	0.797	0.912
4	0.697	0.303	0.000	1	0.207	0.860	0.993
5	0.317	0.013	0.670	3	0.991	1.000	0.984
6	0.032	0.366	0.601	3	0.981	0.978	0.887

---