

## NAG C Library Function Document

### nag\_mann\_whitney (g08amc)

#### 1 Purpose

nag\_mann\_whitney (g08amc) performs the Mann–Whitney  $U$  test on two independent samples of possibly unequal size and calculates the exact probability for the Mann–Whitney rank sum test statistic for the case where there are either ties or no ties in the samples pooled together.

#### 2 Specification

```
#include <nag.h>
#include <nagg08.h>

void nag_mann_whitney (Integer n1, const double x[], Integer n2,
                      const double y[], Nag_TailProbability tail, Nag_CompProb exact,
                      double *u, double *z, double *p, NagError *fail)
```

#### 3 Description

The Mann–Whitney  $U$  test investigates the difference between two populations defined by the distribution functions  $F(x)$  and  $G(y)$  respectively. The data consist of two independent samples of size  $n_1$  and  $n_2$ , denoted by  $x_1, x_2, \dots, x_{n_1}$  and  $y_1, y_2, \dots, y_{n_2}$ , taken from the two populations.

The hypothesis under test,  $H_0$ , often called the null hypothesis, is that the two distributions are the same, that is  $F(x) = G(x)$ , and this is to be tested against an alternative hypothesis  $H_1$  which is

$H_1 : F(x) \neq G(y)$ ; or

$H_1 : F(x) < G(y)$ , i.e., the  $x$ 's tend to be greater than the  $y$ 's; or

$H_1 : F(x) > G(y)$ , i.e., the  $x$ 's tend to be less than the  $y$ 's,

using a two-tailed, upper-tailed or lower-tailed probability respectively. The user selects the alternative hypothesis by choosing the appropriate tail probability to be computed (see the description of argument **tail** in Section 4).

Note that when using this test to test for differences in the distributions one is primarily detecting differences in the location of the two distributions. That is to say, if we reject the null hypothesis  $H_0$  in favour of the alternative hypothesis  $H_1 : F(x) > G(y)$  we have evidence to suggest that the location, of the distribution defined by  $F(x)$ , is less than the location, of the distribution defined by  $G(y)$ .

The Mann–Whitney  $U$  test differs from the Median test (see nag\_median\_test (g08acc)) in that the ranking of the individual scores within the pooled sample is taken into account, rather than simply the position of a score relative to the median of the pooled sample. It is therefore a more powerful test if score differences are meaningful.

The test procedure involves ranking the pooled sample, average ranks being used for ties. Let  $r_{1i}$  be the rank assigned to  $x_i$ , for  $i = 1, 2, \dots, n_1$  and  $r_{2j}$  the rank assigned to  $y_j$ , for  $j = 1, 2, \dots, n_2$ . Then the test statistic  $U$  is defined as follows;

$$U = \sum_{i=1}^{n_1} r_{1i} - \frac{n_1(n_1 + 1)}{2}$$

$U$  is also the number of times a score in the second sample precedes a score in the first sample (where we only count a half if a score in the second sample actually equals a score in the first sample).

nag\_mann\_whitney returns:

- (a) the test statistic  $U$ ;
- (b) the approximate Normal test statistic,

$$z = \frac{U - \text{mean}(U) \pm \frac{1}{2}}{\sqrt{\text{var}(U)}}$$

where

$$\text{mean}(U) = \frac{n_1 n_2}{2}$$

and

$$\text{var}(U) = \frac{n_1 n_2 (n_1 + n_2 + 1)}{12} - \frac{n_1 n_2}{(n_1 + n_2)(n_1 + n_2 - 1)} \times TS$$

where

$$TS = \sum_{j=1}^{\tau} \frac{(t_j)(t_j - 1)(t_j + 1)}{12}$$

$\tau$  is the number of groups of ties in the sample and  $t_j$  is the number of ties in the  $j$ th group.

Note that if no ties are present the variance of  $U$  reduces to  $\frac{n_1 n_2}{12} (n_1 + n_2 + 1)$ .

- (c) An indicator as to whether ties were present in the pooled sample or not.
- (d) The tail probability,  $p$ , corresponding to  $U$ , depending on the choice of **tail**, i.e., the choice of alternative hypothesis,  $H_1$ . An exact probability or a normal approximation may be selected using the **exact** option. For large values of  $n_1$  and  $n_2$  the normal approximation should be adequate while the exact computation may require extensive calculation, particularly in cases where there are ties in the observations. For small samples the exact probability should be used.

The value of  $p$  can be used to perform a significance test on the null hypothesis  $H_0$  against the alternative hypothesis  $H_1$ . Let  $\alpha$  be the size of the significance test (that is,  $\alpha$  is the probability of rejecting  $H_0$  when  $H_0$  is true). If  $p < \alpha$  then the null hypothesis is rejected. Typically  $\alpha$  might be 0.05 or 0.01.

## 4 Parameters

- |    |  |              |
|----|--|--------------|
| 1: | <b>n1</b> – Integer  | <i>Input</i> |
|    | <i>On entry:</i> the number of non-tied pairs, $n_1$ .   |              |
|    | <i>Constraint:</i> <b>n1</b> $\geq$ 1.   |              |
| 2: | <b>x[n1]</b> – const double  | <i>Input</i> |
|    | <i>On entry:</i> the first vector of observations. $x_1, x_2, \dots, x_{n_1}$ .                  |              |
| 3: | <b>n2</b> – Integer  | <i>Input</i> |
|    | <i>On entry:</i> the size of the second sample, $n_2$ .  |              |
|    | <i>Constraint:</i> <b>n2</b> $\geq$ 1.   |              |
| 4: | <b>y[n2]</b> – const double  | <i>Input</i> |
|    | <i>On entry:</i> the second vector of observations. $y_1, y_2, \dots, y_{n_2}$ .                 |              |
| 5: | <b>tail</b> – Nag_TailProbability  | <i>Input</i> |
|    | <i>On entry:</i> indicates the choice of tail probability, and hence the alternative hypothesis. |              |

If **tail** = Nag\_TwoTail, then a two-tailed probability is calculated and the alternative hypothesis is  $H_1 : F(x) \neq G(y)$ .

If **tail** = Nag\_UpperTail, then an upper-tailed probability is calculated and the alternative hypothesis  $H_1 : F(x) < G(y)$ , i.e., the  $x$ 's tend to be greater than the  $y$ 's.

If **tail** = Nag\_LowerTail, then a lower-tailed probability is calculated and the alternative hypothesis  $H_1 : F(x) > G(y)$ , i.e., the  $x$ 's tend to be less than the  $y$ 's.

*Constraint:* **tail** = Nag\_TwoTail, Nag\_UpperTail or Nag\_LowerTail.

- 6: **exact** – Nag\_CompProb *Input*  
*On entry:* indicates if exact probability,  $p$ , is to be computed.  
 If **exact** = Nag\_CompProbExact, then compute the exact probability.  
 If **exact** = Nag\_CompProbApprox, then compute the approximate probability.
- 7: **u** – double \* *Output*  
*On exit:* the Mann–Whitney rank sum statistic,  $U$ .
- 8: **z** – double \* *Output*  
*On exit:* the approximate Normal test statistic,  $z$ , as described in Section 3.
- 9: **p** – double \* *Output*  
*On exit:* the exact tail probability,  $p$ , as specified by the parameter **tail**.
- 10: **fail** – NagError \* *Input/Output*  
 The NAG error parameter (see the Essential Introduction).

## 5 Error Indicators and Warnings

### NE\_INT\_ARG\_LT

On entry, **n1** must not be less than 1: **n1** = *<value>*.

On entry, **n2** must not be less than 1: **n2** = *<value>*.

### NE\_BAD\_PARAM

On entry, parameter **tail** had an illegal value.

On entry, parameter **exact** had an illegal value.

### NE\_MANN\_WHITNEY\_STAT

The computed Mann–Whitney statistic,  $U = \textit{<value>}$ . If the exact tail probability is to be computed, then  $U \geq 0$ .

### NE\_G08AG\_SAMP\_IDEN

The pooled samples are all the same, that is the variance of  $U = 0.0$ .

### NE\_ALLOC\_FAIL

Memory allocation failed.

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

## 6 Further Comments

The time taken by the routine increases with  $n_1$  and  $n_2$ .

## 6.1 Accuracy

The approximate tail probability,  $p$ , returned by `nag_mann_whitney` is a good approximation to the exact probability for cases where  $\max(n_1, n_2) \geq 30$  and  $(n_1 + n_2) \geq 40$ . The relative error of the approximation should be less than 10 percent, for most cases falling in this range.

## 6.2 References

Conover W J (1980) *Practical Nonparametric Statistics* Wiley

Neumann N (1988) Some procedures for calculating the distributions of elementary nonparametric test statistics *Statistical Software Newsletter* **14** (3) 120–126

Siegel S (1956) *Non-parametric Statistics for the Behavioral Sciences* McGraw-Hill

## 7 See Also

`nag_median_test` (g08acc)

## 8 Example

The example program performs the Mann–Whitney test on two independent samples of sizes 16 and 23 respectively. This is used to test the null hypothesis that the distributions of the two populations from which the samples were taken are the same against the alternative hypothesis that the distributions are different. The test statistic, the approximate Normal statistic and the approximate two-tail probability are printed. An exact tail probability is also calculated and printed depending on whether ties were found in the pooled sample or not.

### 8.1 Program Text

```

/* nag_mann_whitney (g08amc) Example Program.
 *
 * Copyright 2000 Numerical Algorithms Group.
 *
 * Mark 6, 2000.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg08.h>

int main (void)
{
    double p, u, z, *x=0, *y=0;
    Integer i, n1, n2;
    Integer exit_status=0;
    NagError fail;

    INIT_FAIL(fail);
    Vprintf("g08amc Example Program Results\n\n");

    /* Skip heading in data file */
    Vscanf("%*[\n]");

    Vscanf("%ld %ld ", &n1, &n2);
    Vprintf("%s%5ld\n", "Sample size of group 1 = ", n1);
    Vprintf("%s%5ld\n", "Sample size of group 2 = ", n2);

```

```

if (!(x = NAG_ALLOC(n1, double))
    || !(y = NAG_ALLOC(n2, double)))
{
    Vprintf("Allocation failure\n");
    exit_status = -1;
    goto END;
}
Vprintf("\n");
for (i = 1; i <= n1; ++i)
    Vscanf("%lf", &x[i - 1]);
Vprintf("%s\n", "Mann-Whitney U test");
Vprintf("\n");
Vprintf("%s\n", "Data values");
Vprintf("\n");
Vprintf("%s", "    Group 1  ");
for (i = 1; i <= n1; ++i)
    Vprintf("%5.1f%s", x[i - 1], i%8?" ":"\n");
for (i = 1; i <= n2; ++i)
    Vscanf("%lf", &y[i - 1]);
Vprintf("\n");
Vprintf("%s", "    Group 2  ");
for (i = 1; i <= n2; ++i)
    Vprintf("%5.1f%s", y[i - 1], i%8?" ":"\n");

g08amc(n1, x, n2, y, Nag_LowerTail, Nag_CompProbApprox,
&u, &z, &p, &fail);
if (fail.code != NE_NOERROR)
{
    Vprintf("Error from g08amc.\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
Vprintf("\n\n");
Vprintf("%s%8.4f\n", "Test statistic          = ", u);
Vprintf("%s%8.4f\n", "Normal Statistic          = ", z);
Vprintf("%s%8.4f\n", "Approximate tail probability = ", p);
g08amc(n1, x, n2, y, Nag_LowerTail, Nag_CompProbExact,
&u, &z, &p, &fail);
if (fail.code != NE_NOERROR)
{
    Vprintf("Error from g08amc.\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}
Vprintf("%s%8.4f\n", "Exact tail probability          = ", p);
END:
if(x) NAG_FREE(x);
if(y) NAG_FREE(y);
return exit_status;
}

```

## 8.2 Program Data

g08amc Example Program Data

16 23

13.0 6.0 12.0 7.0 12.0 7.0 10.0 7.0

```
10.0  7.0 16.0  7.0 10.0  8.0  9.0  8.0
17.0  6.0 10.0  8.0 15.0  8.0 15.0 10.0 15.0 10.0 14.0 10.0
14.0 11.0 14.0 11.0 13.0 12.0 13.0 12.0 13.0 12.0 12.0
```

### 8.3 Program Results

g08amc Example Program Results

```
Sample size of group 1 =    16
Sample size of group 2 =    23
```

Mann-Whitney U test

Data values

```
Group 1  13.0  6.0 12.0  7.0 12.0  7.0 10.0  7.0
          10.0  7.0 16.0  7.0 10.0  8.0  9.0  8.0

Group 2  17.0  6.0 10.0  8.0 15.0  8.0 15.0 10.0
          15.0 10.0 14.0 10.0 14.0 11.0 14.0 11.0
          13.0 12.0 13.0 12.0 13.0 12.0 12.0
```

```
Test statistic           =  86.0000
Normal Statistic        = -2.8039
Approximate tail probability =  0.0025
Exact tail probability   =  0.0020
```

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