

## NAG C Library Function Document

### nag\_tsa\_cross\_corr (g13bcc)

#### 1 Purpose

nag\_tsa\_cross\_corr (g13bcc) calculates cross-correlations between two time series.

#### 2 Specification

```
void nag_tsa_cross_corr (const double x[], const double y[], Integer nxy,
    Integer nl, double *s, double *r0, double r[], double *stat, NagError *fail)
```

#### 3 Description

Given two series  $x_1, x_2, \dots, x_n$  and  $y_1, y_2, \dots, y_n$  the routine calculates the cross-correlations between  $x_t$  and lagged values of  $y_t$ :

$$r_{xy}(l) = \frac{\sum_{t=1}^{n-l} (x_t - \bar{x})(y_{t+l} - \bar{y})}{n s_x s_y}, \quad l = 0, 1, \dots, L$$

where

$$\bar{x} = \frac{\sum_{t=1}^n x_t}{n}$$

$$s_x^2 = \frac{\sum_{t=1}^n (x_t - \bar{x})^2}{n}$$

and similarly for  $y$ .

The ratio of standard deviations  $s_y/s_x$  is also returned, and a portmanteau statistic is calculated:

$$\mathbf{stat} = n \sum_{l=1}^L r_{xy}(l)^2.$$

Provided  $n$  is large,  $L$  much less than  $n$ , and both  $x_t, y_t$  are samples of series whose true autocorrelation functions are zero, then, under the null hypothesis that the true cross-correlations between the series are zero, **stat** has a  $\chi^2$  distribution with  $L$  degrees of freedom. Values of **stat** in the upper tail of this distribution provide evidence against the null hypothesis.

#### 4 References

Box G E P and Jenkins G M (1976) *Time Series Analysis: Forecasting and Control* (Revised Edition) Holden-Day

#### 5 Parameters

- |    |  |              |
|----|--|--------------|
| 1: | <b>x[nxy]</b> – const double                       | <i>Input</i> |
|    | <i>On entry:</i> the $n$ values of the $x$ series. |              |
| 2: | <b>y[nxy]</b> – const double                       | <i>Input</i> |
|    | <i>On entry:</i> the $n$ values of the $y$ series. |              |

- 3: **nxy** – Integer *Input*  
*On entry:* the length of the time series,  $n$ .  
*Constraint:*  $\mathbf{nxy} \geq 2$ .
- 4: **nl** – Integer *Input*  
*On entry:* the maximum lag for calculating cross-correlations,  $L$ .  
*Constraint:*  $1 \leq \mathbf{nl} < \mathbf{nxy}$ .
- 5: **s** – double \* *Output*  
*On exit:* the ratio of the standard deviation of the  $y$  series to the standard deviation of the  $x$  series,  $s_y/s_x$ .
- 6: **r0** – double \* *Output*  
*On exit:* the cross-correlation between the  $x$  and  $y$  series at lag zero.
- 7: **r[nl]** – double *Output*  
*On exit:* **r**[ $l - 1$ ] contains the cross-correlations between the  $x$  and  $y$  series at lags  $l$ ,  $r_{xy}(l)$ , for  $l = 1, 2, \dots, L$ .
- 8: **stat** – double \* *Output*  
*On exit:* the statistic for testing for absence of cross-correlation.
- 9: **fail** – NagError \* *Input/Output*  
The NAG error parameter (see the Essential Introduction).

## 6 Error Indicators and Warnings

### NE\_INT

On entry, **nl** =  $\langle value \rangle$ .  
Constraint: **nl** > 1.

On entry, **nxy** =  $\langle value \rangle$ .  
Constraint: **nxy** > 1.

### NE\_INT\_2

On entry, **nl**  $\geq$  **nxy**: **nl** =  $\langle value \rangle$ , **nxy** =  $\langle value \rangle$ .

### NE\_ZERO\_VARIANCE

One or both of the  $x$  and  $y$  series have zero variance.

### NE\_BAD\_PARAM

On entry, parameter  $\langle value \rangle$  had an illegal value.

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

## 7 Accuracy

All computations are believed to be stable.

## 8 Further Comments

The time taken by the routine is approximately proportional to  $nL$ .

## 9 Example

The example program reads two time series of length 20. It calculates and prints the cross-correlations up to lag 15 for the first series leading the second series and then for the second series leading the first series.

### 9.1 Program Text

```

/* nag_tsa_cross_corr (g13bcc) Example Program.
 *
 * Copyright 2002 Numerical Algorithms Group.
 *
 * Mark 7, 2002.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg13.h>

int main(void)
{
    /* Scalars */
    double r0xy, r0yx, statxy, statyx, sxy, syx;
    Integer exit_status, i, nl, nxy;
    NagError fail;

    /* Arrays */
    double *rxy = 0, *ryx = 0, *x = 0, *y = 0;

    INIT_FAIL(fail);
    exit_status = 0;

    Vprintf("g13bcc Example Program Results\n");

    /* Skip heading in data file */
    Vscanf("%*[\n] ");

    /* Read series length and number of lags */
    Vscanf("%ld%ld%*[\n] ", &nxy, &nl);

    /* Allocate memory */
    if ( !(rxy = NAG_ALLOC(nl, double)) ||
         !(ryx = NAG_ALLOC(nl, double)) ||
         !(x = NAG_ALLOC(nxy, double)) ||
         !(y = NAG_ALLOC(nxy, double)) )
    {
        Vprintf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }

    /* Read series */
    for (i = 1; i <= nxy; ++i)
        Vscanf("%lf", &x[i-1]);
    Vscanf("%*[\n] ");

    for (i = 1; i <= nxy; ++i)
        Vscanf("%lf", &y[i-1]);
    Vscanf("%*[\n] ");

    /* Call routine to calculate cross correlations between X and Y */
    g13bcc(x, y, nxy, nl, &sxy, &r0xy, rxy, &statxy, &fail);
    if (fail.code != NE_NOERROR)
    {

```

```

        Vprintf("Error from g13bcc, 1st call.\n%s\n", fail.message);
        exit_status = 1;
        goto END;
    }

    /* Call routine to calculate cross correlations between Y and X */
    g13bcc(y, x, nxy, nl, &syx, &r0yx, ryx, &statyx, &fail);
    if (fail.code != NE_NOERROR)
    {
        Vprintf("Error from g13bcc, 2nd call.\n%s\n", fail.message);
        exit_status = 1;
        goto END;
    }

    Vprintf("\n");
    Vprintf("          Between          Between\n");
    Vprintf("          X and Y          Y and X\n");
    Vprintf("\n");
    Vprintf("Standard deviation ratio%10.4f%15.4f\n", sxy, syx);
    Vprintf("\n");
    Vprintf("Cross correlation at lag\n");
    Vprintf("          0\n");
    Vprintf("%10.4f%15.4f\n", r0xy, r0yx);
    for (i = 1; i <= nl; ++i)
        Vprintf("          %4ld%10.4f%15.4f\n", i, rxy[i-1], ryx[i-
1]);
    Vprintf("\n");
    Vprintf("Test statistic          %10.4f%15.4f\n", statxy, statyx);

END:
    if (rxy) NAG_FREE(rxy);
    if (ryx) NAG_FREE(ryx);
    if (x) NAG_FREE(x);
    if (y) NAG_FREE(y);

    return exit_status;
}

```

## 9.2 Program Data

g13bcc Example Program Data

20	15									
0.02	0.05	0.08	0.03	-0.05	0.11	-0.01	-0.08	-0.08	-0.11	
-0.18	-0.19	-0.09	0.03	0.10	0.15	-0.14	0.07	0.09	0.16	
3.18	3.21	3.26	3.25	3.08	3.01	3.06	3.17	3.12	3.04	
3.26	3.45	3.33	3.70	3.31	3.81	3.33	2.96	3.28	3.10	

## 9.3 Program Results

g13bcc Example Program Results

	Between X and Y	Between Y and X
Standard deviation ratio	2.0053	0.4987
Cross correlation at lag		
0	0.0568	0.0568
1	0.0438	-0.0151
2	-0.3762	0.3955
3	-0.4864	0.3417
4	-0.6294	0.5486
5	-0.3871	0.2291
6	-0.1690	0.3190
7	-0.0678	0.1980
8	0.0962	0.0438
9	0.0788	-0.1428
10	0.2910	-0.1376

	11	0.0950	-0.0387
	12	0.0547	-0.0380
	13	0.1855	-0.1551
	14	0.0243	-0.1536
	15	0.0034	-0.0696
Test statistic		22.1269	17.2917

---