NAG C Library Function Document

nag_tsa_spectrum_univar_cov (g13cac)

1   Purpose

nag_tsa_spectrum_univar_cov (g13cac) calculates the smoothed sample spectrum of a univariate time series using one of four lag windows – rectangular, Bartlett, Tukey or Parzen window.

2   Specification

```
void nag_tsa_spectrum_univar_cov (Integer nx, Integer mtx, double px, Integer iw,
    Integer mw, Integer ic, Integer nc, double c[], Integer kc, Integer l,
    Nag_LoggedSpectra lg_spect, Integer nxg, double xg[], Integer *ng,
    double stats[], NagError *fail)
```

3   Description

The smoothed sample spectrum is defined as

$$f(\omega) = \frac{1}{2\pi}\left(C_0 + 2\sum k = 1 M - 1 w_k C_k \cos(\omega k)\right),$$

where M is the window width, and is calculated for frequency values

$$\omega_i = \frac{2\pi i}{L}, \quad i = 0, 1, \ldots, [L/2],$$

where $[\ ]$ denotes the integer part.

The autocovariances C_k may be supplied by the user, or constructed from a time series $x_1, x_2, \ldots, x_n$, as

$$C_k = \frac{1}{n}\sum t = 1 n - k x_t x_{t+k},$$

the fast Fourier transform (FFT) being used to carry out the convolution in this formula.

The time series may be mean or trend corrected (by classical least squares), and tapered before calculation of the covariances, the tapering factors being those of the split cosine bell:

$$\tfrac{1}{2}(1 - \cos(\pi(t - \tfrac{1}{2})/T)), \qquad 1 \le t \le T$$

$$\tfrac{1}{2}(1 - \cos(\pi(n - t + \tfrac{1}{2})/T)), \quad n + 1 - T \le t \le n$$

$$1, \qquad\qquad \text{otherwise,}$$

where $T = \left[\frac{np}{2}\right]$ and p is the tapering proportion.

The smoothing window is defined by

$$w_k = W\left(\frac{k}{M}\right), \quad k \le M - 1,$$

which for the various windows is defined over $0 \le \alpha < 1$ by

rectangular:

$$W(\alpha) = 1$$

Bartlett:

$$W(\alpha) = 1 - \alpha$$

Tukey:

$$W(\alpha) = \frac{1}{2}(1 + \cos(\pi\alpha))$$

Parzen:

$$W(\alpha) = 1 - 6\alpha^2 + 6\alpha^3, \quad 0 \le \alpha \le \tfrac{1}{2}$$

$$W(\alpha) = 2(1 - \alpha)^3, \qquad \tfrac{1}{2} < \alpha < 1.$$

The sampling distribution of $f(\omega)$ is approximately that of a scaled χ_d^2 variate, whose degrees of freedom d is provided by the routine, together with multiplying limits mu, ml from which approximate 95% confidence intervals for the true spectrum $f(\omega)$ may be constructed as $[ml \times f(\omega), \ mu \times f(\omega)]$. Alternatively, $\log f(\omega)$ may be returned, with additive limits.

The bandwidth b of the corresponding smoothing window in the frequency domain is also provided. Spectrum estimates separated by (angular) frequencies much greater than b may be assumed to be independent.

4 References

Jenkins G M and Watts D G (1968) *Spectral Analysis and its Applications* Holden–Day

Bloomfield P (1976) *Fourier Analysis of Time Series: An Introduction* Wiley

5 Parameters

1:   **nx** – Integer   *Input*

On entry: the length of the time series, n.

Constraint: **nx** ≥ 1.

2:   **mtx** – Integer   *Input*

On entry: if covariances are to be calculated by the routine (**ic** $= 0$), **mtx** must specify whether the data are to be initially mean or trend corrected.

mtx $= 0$

> For no correction.

mtx $= 1$

> For mean correction.

mtx $= 2$

> For trend correction.

Constraints:

> if **ic** $= 0$, $0 \le$ **mtx** ≤ 2.

If covariances are supplied (**ic** $\ne 0$), **mtx** is not used.

3:   **px** – double   *Input*

On entry: if covariances are to be calculated by the routine (**ic** $= 0$), **px** must specify the proportion of the data (totalled over both ends) to be initially tapered by the split cosine bell taper. If covariances are supplied (**ic** $\ne 0$), then **px** must specify the proportion of data tapered before the supplied covariances were calculated and after any mean or trend correction. **px** is required for the calculation of output statistics. A value of 0.0 implies no tapering.

Constraint: $0.0 \le$ **px** ≤ 1.0.

4:     **iw** – Integer   *Input*

   On entry: the choice of lag window. **iw** = 1 for rectangular, 2 for Bartlett, 3 for Tukey or 4 for Parzen.

   Constraint: $1 \leq \mathbf{iw} \leq 4$.

5:     **mw** – Integer   *Input*

   On entry: the 'cut-off' point, M, of the lag window. Windowed covariances at lag M or greater are zero.

   Constraint: $1 \leq \mathbf{mw} \leq \mathbf{nx}$.

6:     **ic** – Integer   *Input*

   On entry: indicates whether covariances are to be calculated in the routine or supplied in the call to the routine.

   ic = 0

      Covariances are to be calculated.

   ic $\neq$ 0

      Covariances are to be supplied.

7:     **nc** – Integer   *Input*

   On entry: the number of covariances to be calculated in the routine or supplied in the call to the routine.

   Constraint: $\mathbf{mw} \leq \mathbf{nc} \leq \mathbf{nx}$.

8:     **c**[**nc**] – double   *Input/Output*

   On entry: if **ic** $\neq$ 0, then **c** must contain the **nc** covariances for lags from 0 to (**nc** − 1), otherwise **c** need not be set.

   On exit: if **ic** = 0, **c** will contain the **nc** calculated covariances.

   If **ic** $\neq$ 0, the contents of **c** will be unchanged.

9:     **kc** – Integer   *Input*

   On entry: if **ic** = 0, **kc** must specify the order of the fast Fourier transform (FFT) used to calculate the covariances. **kc** should be a product of small primes such as 2^m where m is the smallest integer such that $2^m \geq \mathbf{nx} + \mathbf{nc}$, provided $m \leq 20$.

   If **ic** $\neq$ 0, that is covariances are supplied, then **kc** is not used.

   Constraint: $\mathbf{kc} \geq \mathbf{nx} + \mathbf{nc}$. The largest prime factor of **kc** must not exceed 19, and the total number of prime factors of **kc**, counting repetitions, must not exceed 20.

10:    **l** – Integer   *Input*

   On entry: the frequency division, L, of the spectral estimates as $\frac{2\pi}{L}$. Therefore it is also the order of the FFT used to construct the sample spectrum from the covariances. **l** should be a product of small primes such as 2^m where m is the smallest integer such that $2^m \geq 2M - 1$, provided $m \leq 20$.

   Constraint: $\mathbf{l} \geq 2 \times \mathbf{mw} - 1$. The largest prime factor of **l** must not exceed 19, and the total number of prime factors of **l**, counting repetitions, must not exceed 20.

11:    **lg_spect** – Nag_LoggedSpectra   *Input*

   On entry: indicates whether unlogged or logged spectral estimates and confidence limits are required.

lg_spect = **Nag_Unlogged**

> Unlogged.

lg_spect = **Nag_Logged**

> Logged.

Constraint: **lg_spect** = **Nag_Unlogged** or **Nag_Logged**.

12: **nxg** – Integer   *Input*

On entry: the dimension of the array **xg** as declared in the function from which nag_tsa_spectrum_univar_cov (g13cac) is called.

Constraints:

> if **ic** = 0, **nxg** ≥ max(**kc**, **l**);;
> if **ic** ≠ 0, **nxg** ≥ **l**.

13: **xg**[**nxg**] – double   *Input/Output*

On entry: if the covariances are to be calculated, then **xg** must contain the **nx** data points. If covariances are supplied, **xg** may contain any values.

On exit: contains the **ng** spectral estimates, $f(\omega_i)$, for $i = 0, 1, \ldots, [L/2]$ in **xg**[0] to **xg**[**ng** − 1] respectively (logged if **lg_spect** = **Nag_Logged**). The elements **xg**[i − 1], for i = **ng** + 1, . . . , **nxg** contain 0.0.

14: **ng** – Integer *   *Output*

On exit: the number of spectral estimates, $[L/2] + 1$, in **xg**.

15: **stats**[4] – double   *Output*

On exit: four associated statistics. These are the degrees of freedom in **stats**[0], the lower and upper 95% confidence limit factors in **stats**[1] and **stats**[2] respectively (logged if **lg_spect** = **Nag_Logged**), and the bandwidth in **stats**[3].

16: **fail** – NagError *   *Input/Output*

The NAG error parameter (see the Essential Introduction).

6   Error Indicators and Warnings

NE_INT

> On entry, **mw** = ⟨*value*⟩.
> Constraint: **mw** ≥ 1.
>
> On entry, **iw** is not equal to 1, 2, 3 or 4: **iw** = ⟨*value*⟩.
>
> On entry, **ic** = 0 and **mtx** > 2: **mtx** = ⟨*value*⟩.
>
> On entry, **ic** = 0 and **mtx** < 0: **mtx** = ⟨*value*⟩.
>
> On entry, **nx** = ⟨*value*⟩.
> Constraint: **nx** ≥ 1.

NE_INT_2

> On entry, **mw** > **nx**: **mw** = ⟨*value*⟩, **nx** = ⟨*value*⟩.
>
> On entry, **l** < 2 × **mw** − 1: **l** = ⟨*value*⟩, **mw** = ⟨*value*⟩.
>
> On entry, **ic** is not equal to 0 and **nxg** < **l**: **nxg** = ⟨*value*⟩, **l** = ⟨*value*⟩.
>
> On entry, **nc** > **nx**: **nc** = ⟨*value*⟩, **nx** = ⟨*value*⟩.

On entry, **nc** < **mw**: **nc** = ⟨*value*⟩, **mw** = ⟨*value*⟩.

NE_INT_3

On entry, **ic** = 0 and **kc** < (**nx** + **nc**): **kc** = ⟨*value*⟩, **nx** = ⟨*value*⟩, **nc** = ⟨*value*⟩.

On entry, **ic** = 0 and **nxg** < max(**kc,l**): **nxg** = ⟨*value*⟩, **kc** = ⟨*value*⟩, **l** = ⟨*value*⟩.

NE_CONFID_LIMITS

The calculation of confidence limit factors has failed.

NE_PRIME_FACTOR

l has a prime factor exceeding 19, or more than 20 prime factors (counting repetitions): **l** = ⟨*value*⟩.

kc has a prime factor exceeding 19, or more than 20 prime factors (counting repetitions): **kc** = ⟨*value*⟩.

NE_REAL

On entry, **px** = ⟨*value*⟩.
Constraint: **px** ≤ 1.0.

On entry, **px** = ⟨*value*⟩.
Constraint: **px** ≥ 0.0.

NE_SPECTRAL_ESTIMATES

One or more spectral estimates are zero. Consult the values in **xg** and **stats**.

NE_BAD_PARAM

On entry, parameter ⟨*value*⟩ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

The FFT is a numerically stable process, and any errors introduced during the computation will normally be insignificant compared with uncertainty in the data.

8 Further Comments

nag_tsa_spectrum_univar_cov (g13cac) carries out two FFTs of length **kc** to calculate the covariances and one FFT of length **l** to calculate the sample spectrum. The time taken by the routine for an FFT of length n is approximately proportional to $n \log(n)$.

9 Example

The example program reads a time series of length 256. It selects the mean correction option, a tapering proportion of 0.1, the Parzen smoothing window and a cut-off point for the window at lag 100. It chooses to have 100 auto-covariances calculated and unlogged spectral estimates at a frequency division of $2\pi/200$. It then calls nag_tsa_spectrum_univar_cov (g13cac) to calculate the univariate spectrum and statistics and prints the autocovariances and the spectrum together with its 95% confidence multiplying limits.

9.1 Program Text

```c
/* nag_tsa_spectrum_univar_cov (g13cac) Example Program.
 *
 * Copyright 2002 Numerical Algorithms Group.
 *
 * Mark 7, 2002.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg13.h>

int main(void)
{
  /* Scalars */
  double px;
  Integer exit_status, i, ic, iw, kc, lf, lg, mtx, mw, nc, ng, nx, nxg;
  NagError fail;

  /* Arrays */
  double *c = 0, *xg = 0;
  double stats[4];

  INIT_FAIL(fail);
  exit_status = 0;

  Vprintf("g13cac Example Program Results\n");

  /* Skip heading in data file */
  Vscanf("%*[^\n] ");

  Vscanf("%ld%ld%*[^\n] ", &nx, &nc);

  if (nx > 0 && nc > 0)
    {
      mtx = 1;
      px = 0.1;
      iw = 4;
      mw = 100;
      ic = 0;
      kc = 360;
      lf = 200;
      lg = 0;

      if (ic == 0)
        nxg = MAX(kc, lf);
      else
        nxg = lf;

      /* Allocate memory */
      if ( !(c = NAG_ALLOC(nc, double)) ||
           !(xg = NAG_ALLOC(nxg, double)) )
        {
          Vprintf("Allocation failure\n");
          exit_status = -1;
          goto END;
        }

      for (i = 1; i <= nx; ++i)
        Vscanf("%lf", &xg[i-1]);
      Vscanf("%*[^\n] ");

      g13cac(nx, mtx, px, iw, mw, ic, nc, c, kc, lf, Nag_Unlogged,
             nxg, xg, &ng, stats, &fail);
      if (fail.code != NE_NOERROR)
        {
          Vprintf("Error from g13cac.\n%s\n",   fail.message);
          exit_status = 1;
          goto END;
```

```
        }

    Vprintf("\n");

        Vprintf("Covariances\n");
        for (i = 1; i <= nc; ++i)
          {
            Vprintf("%11.4f", c[i-1]);
            if (i % 6 == 0 || i == nc)
              Vprintf("\n");
          }
        Vprintf("\n");
        Vprintf("Degrees of freedom =%4.1f      Bandwidth =%7.4f\n",
            stats[0], stats[3]);
        Vprintf("\n");
        Vprintf("95 percent confidence limits -     Lower =%7.4f  "
            "Upper =%7.4f\n", stats[1], stats[2]);
        Vprintf("\n");
        Vprintf("      Spectrum      Spectrum      Spectrum"
            "      Spectrum\n");
        Vprintf("      estimate      estimate      estimate"
            "      estimate\n");
        for (i = 1; i <= ng; ++i)
          {
            Vprintf("%4ld%10.4f", i, xg[i-1]);
            if (i % 4 == 0 || i == ng)
              Vprintf("\n");
          }

    }

END:
  if (c) NAG_FREE(c);
  if (xg) NAG_FREE(xg);

  return exit_status;
}
```

9.2 Program Data

```
g13cac Example Program Data
 256 100
   5.0   11.0   16.0   23.0   36.0   58.0   29.0   20.0   10.0    8.0    3.0    0.0
   0.0    2.0   11.0   27.0   47.0   63.0   60.0   39.0   28.0   26.0   22.0   11.0
  21.0   40.0   78.0  122.0  103.0   73.0   47.0   35.0   11.0    5.0   16.0   34.0
  70.0   81.0  111.0  101.0   73.0   40.0   20.0   16.0    5.0   11.0   22.0   40.0
  60.0   80.9   83.4   47.7   47.8   30.7   12.2    9.6   10.2   32.4   47.6   54.0
  62.9   85.9   61.2   45.1   36.4   20.9   11.4   37.8   69.8  106.1  100.8   81.6
  66.5   34.8   30.6    7.0   19.8   92.5  154.4  125.9   84.8   68.1   38.5   22.8
  10.2   24.1   82.9  132.0  130.9  118.1   89.9   66.6   60.0   46.9   41.0   21.3
  16.0    6.4    4.1    6.8   14.5   34.0   45.0   43.1   47.5   42.2   28.1   10.1
   8.1    2.5    0.0    1.4    5.0   12.2   13.9   35.4   45.8   41.1   30.1   23.9
  15.6    6.6    4.0    1.8    8.5   16.6   36.3   49.6   64.2   67.0   70.9   47.8
  27.5    8.5   13.2   56.9  121.5  138.3  103.2   85.7   64.6   36.7   24.2   10.7
  15.0   40.1   61.5   98.5  124.7   96.3   66.6   64.5   54.1   39.0   20.6    6.7
   4.3   22.7   54.8   93.8   95.8   77.2   59.1   44.0   47.0   30.5   16.3    7.3
  37.6   74.0  139.0  111.2  101.6   66.2   44.7   17.0   11.3   12.4    3.4    6.0
  32.3   54.3   59.7   63.7   63.5   52.2   25.4   13.1    6.8    6.3    7.1   35.6
  73.0   85.1   78.0   64.0   41.8   26.2   26.7   12.1    9.5    2.7    5.0   24.4
  42.0   63.5   53.8   62.0   48.5   43.9   18.6    5.7    3.6    1.4    9.6   47.4
  57.1  103.9   80.6   63.6   37.6   26.1   14.2    5.8   16.7   44.3   63.9   69.0
  77.8   64.9   35.7   21.2   11.1    5.7    8.7   36.1   79.7  114.4  109.6   88.8
  67.8   47.5   30.6   16.3    9.6   33.2   92.6  151.6  136.3  134.7   83.9   69.4
  31.5   13.9    4.4   38.0
```

9.3   Program Results

```
g13cac Example Program Results

Covariances
  1152.9733    937.3289    494.9243     14.8648   -342.8548   -514.6479
  -469.2733   -236.6896    109.0608    441.3498    637.4571    641.9954
   454.0505    154.5960   -136.8016   -343.3911   -421.8441   -374.4095
  -241.1943    -55.6140    129.4067    267.4248    311.8293    230.2807
    56.4402   -146.4689   -320.9948   -406.4077   -375.6384   -273.5936
  -132.6214     11.0791    126.4843    171.3391    122.6284    -11.5482
  -169.2623   -285.2358   -331.4567   -302.2945   -215.4832   -107.8732
    -3.4126     73.2521     98.0831     71.8949     17.0985    -27.5632
   -76.7900   -110.5354   -126.1383   -121.1043   -103.9362    -67.4619
   -10.8678     58.5009    116.4587    140.0961    129.5928     66.3211
   -35.5487   -135.3894   -203.7149   -216.2161   -152.7723    -30.4361
    99.3397    188.9594    204.9047    148.4056     34.4975   -103.7840
  -208.5982   -252.4128   -223.7600   -120.8640     23.3565    156.0956
   227.7642    228.5123    172.3820     87.4911    -21.2170   -117.5282
  -176.3634   -165.1218    -75.1308     67.1634    195.7290    279.3039
   290.8258    225.3811    104.0784    -44.4731   -162.7355   -207.7480
  -165.2444    -48.5473    118.8872    265.0045

Degrees of freedom = 9.0      Bandwidth = 0.1165

95 percent confidence limits -     Lower = 0.4731  Upper = 3.3329

       Spectrum        Spectrum        Spectrum        Spectrum
       estimate        estimate        estimate        estimate
   1   210.4696    2   428.2020    3   810.1419    4   922.5900
   5   706.1605    6   393.4052    7   207.6481    8   179.0657
   9   170.1320   10   133.0442   11   103.6752   12   103.0644
  13   141.5173   14   194.3041   15   266.5730   16   437.0181
  17   985.3130   18  2023.1574   19  2681.8980   20  2363.7439
  21  1669.9001   22  1012.1320   23   561.4822   24   467.2741
  25   441.9977   26   300.1985   27   172.0184   28   114.7823
  29    79.1533   30    49.4882   31    27.0902   32    16.8081
  33    27.5111   34    59.4429   35    97.0145   36   119.3664
  37   116.6737   38    87.3142   39    54.9570   40    42.9781
  41    46.6097   42    53.6206   43    50.6050   44    36.7780
  45    25.6285   46    24.8555   47    30.2626   48    31.5642
  49    27.3351   50    22.4443   51    18.5418   52    15.2425
  53    12.0207   54    12.6846   55    18.3975   56    19.3058
  57    12.6103   58     7.9511   59     7.1333   60     5.4996
  61     3.4182   62     3.2359   63     5.3836   64     8.5225
  65    10.0610   66     7.9483   67     4.2261   68     3.2631
  69     5.5751   70     7.8491   71     9.3694   72    11.0791
  73    10.1386   74     6.3158   75     3.6375   76     2.6561
  77     1.8026   78     1.0103   79     1.0693   80     2.3950
  81     4.0822   82     4.6221   83     4.0672   84     3.8460
  85     4.8489   86     6.3964   87     6.4762   88     4.9457
  89     4.4444   90     5.2131   91     5.0389   92     4.6141
  93     5.8722   94     7.9268   95     7.9486   96     5.7854
  97     4.5495   98     5.2696   99     6.3893  100     6.5216
 101     6.2129
```